YEAR: - 2010
SUB: - MATH (sub)   F.M:- 100
BASED ON MEMORY

Section-I (Compulsory) (marks: 10)

Each part carries 1 mark

1. Write the *correct* answer in the following:

(a) If $y = e^{mx}$; then $\frac{d^n y}{dx^n}$ is equal to:

(i) me^{mx} (ii) ne^{mx} (iii) $n^m e^{mx}$ (iv) $m^n e^{mx}$

(b) The curvature of the circle $x^2 + y^2 = a^2$ is :

(i) a^2 (ii) a (iii) $1/a$ (iv) $1/a^2$

(c) $\int logx\, dx$ is equal to:

(i) $\frac{1}{x}(logx - 1) + c$ (ii) $x(log\frac{1}{x} - 1) + c$

(iii) $(logx - x) + c$ (iv) $x(logx - 1) + c$

(d) $\int_0^{\pi/4} tanx\, dx$ is equal to :

(i) $\pi/4$ (ii) 1 (iii) $\frac{1}{2}\log 2$ (iv) $2\log\frac{1}{2}$

(e) If $[\vec{a}\,\vec{b}\,\vec{c}] = 0$ then:

(i) All vectors are equal (ii) The vectors are coplanar

(iii) The vectors are mutually perpendicular

(iv) The vectors are parallel to each other

(f) If ϕ be a scalar function then $\nabla \times (\nabla\emptyset)$ is equal to:

(i) 0 (ii) 1 (iii) -1 (iv) $\vec{0}$

(g) The number of normals can in general be drawn to a parabola from any point is :

(i) 1 (ii) 2 (iii) 3 (iv) 4

(h) The polar equation of a conic is :

(i) $r = l(1 - e\,cos\theta)$ (ii) $r = l(1 + e\,cos\theta)$

(iii) $l = r(1 - e\,cos\theta)$ (iv) $l = r(1 + e\,cos\theta)$

(*i*) Sequence $\{U_n\}$ is said to be a strictly monotonic increasing if $\forall$ n $\in$ N :

(i) $U_{n+1} \geq U_n$ (ii) $U_{n+1} \leq U_n$ (iii) $U_{n+1} < U_n$ (iv) $U_{n+1} > U_n$

(j) The series $\sum \frac{n(n+1)}{(n+2)^2}$ is :

(i) Convergent (ii) Conditionally convergent

(iii) Divergent (iv) Oscillatory

Section-II (marks: 90)

Each question has two part (a) carries 2 marks and part (b) carries 7 marks.
Answer ten questions, selecting at least one from each Group.

Group-A

2. (a) If y=log (1+x) , then find y_n .

(b) If $y = \sin^{-1}x$ then prove that following:

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$$

3. (a) Expand log(1+x) by the help of Maclaurin's Theorem.

(b) If $u = \tan^{-1}(y/x)$, then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$

4. (a) Find the equation of the tangent at (a,b) to the curve $(x/a)^n + (y/b)^n = 2.$

(b) In the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that the radius of curvature at the end of the major axis is equal to the semi-latus rectum of the ellipse.

5. (a) If y=x/logx , then find the min. or max. value of y.

(b) Find the asymptotes of the following curve:

$$y^3 + x^2y + 2xy^2 - y + 1 = 0$$

Group-B

6. (a) Integrate : $\int \frac{(t^2-1)}{(t^4+1)} dt$

(b) Integrate : $\int \frac{z\,dz}{(1+z)(1+z^2)}$

7. (a) Prove : $\int_0^a f(\emptyset)d\emptyset = \int_0^a f(a - \emptyset)d\emptyset$

(b) Evaluate : $\int_0^{\pi/4} \tan^5\Psi\, d\Psi$

8. (a) Find the perimeter of the following curve:

$$x^{2/3} + y^{2/3} = a^{2/3}$$

(b) Find the whole area of the curve r=3+2cosθ .

9. (a) Find the volume of the solid formed by the revolution of the ellipse $x^2/a^2 + y^2/b^2 = 1$ about the x-axis.

(b) The cardiode r=a(1+cosθ) revolves about the initial line. Find the surface area of the figure formed.

Group-C

10. (a) If $\vec{r} = \vec{a}\cos wt + \vec{b}\sin wt$, then show that

$$\vec{r} \times \frac{d\vec{r}}{dt} = w\vec{a} \times \vec{b}$$

(b) If $\vec{a}$ is a unit vector, then prove the following :

$$\left|\vec{a} \times \frac{d\vec{a}}{dt}\right| = \left|\frac{d\vec{a}}{dt}\right|.$$

11. (a) Prove : Grad(ΦΨ) Φ gradΨ+ Ψ grad Φ.

(b) Prove :$\nabla.(\vec{a} \times \vec{b}) = \vec{b}.(\nabla \times \vec{a}) - \vec{a}.(\nabla \times \vec{b})$

12. (a) Prove that : $\nabla.(\nabla \times \vec{v}) = 0$; where v have continuous partial derivative.

(b) If $\vec{r}$ =(x,y,z) , then find the value of (i) $\nabla.\vec{r}$ and (ii) $\nabla \times \vec{r}$; when x,y and z are independent of each other.

Group-D

13. (a) By transforming the origin to the point (2,3) and turning the axes through an angle Π/4, find the transformed from of the equation $3x^2+2xy+3y^2-18x-22y+50=0.$

(b) What conic section is represented by $2x^2+3y^2-4x-12y+13=0$? Find the centre, axes and eccentricity of the conic.

14. (a) Find the equation of the normal to the ellipse $x^2/a^2+y^2/b^2=1$ at pt. (x_1,y_1).

(b) Find the condition under which the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ will be represent an ellipse.

15.(a)Find the equation of the cord of contact of tangents drown from the points (x_1,y_1) to the parabola $y^2=4ax$.

(b) Find the equation of the pair of tangents from (1,1) to the ellipse, $2x^2+y^2-4x+2y+2=0$.

16. (a) Prove that the semi-latus rectum of conic $l/r = (1+e\cos\theta)$ is a Harmonic Mean between the segments of the focal cord.

(b) If the polar of (h_1,k_1) and (h_2,k_2) with respect to the hyperbola $x^2/a^2-y^2/b^2=1$ are at the right angles, then prove that $a^4k_1k_2+b^4h_1h_2=0$.

Group-E

17. (a) Define least upper bound and and greatest lower bound of a non-empty subset of real numbers with suitable example.

(b) Every monotonic decreasing sequence tends to its greatest lower bound.

18. (a) Prove that a convergent sequence determine its limit uniquely.

(b) Show, with the help of Cauchy's general principle of convergence, that the sequence f where

$f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}$ is not convergent.

19. (a)If an infinite series $\sum U_n$ is convergent, then $U_n \to 0$ as $n \to \infty$.

(b) Whether series $1 + \frac{2^p}{2!} + \frac{3^p}{3!} + \frac{4^p}{4!}$..... to ∞ for all values of p is convergent or divergent ?

YEAR: - 2011 by pankaj

SUB: - MATH (sub) F.M:- 100

Section-I (Compulsory) (marks: 10)

Each part carries 1 mark

1. Write the *correct* answer in the following:

(a) $y = a^{mx}$, then y_n is

(i) $(m\log_e a)^n . a^{mx}$ (ii) $\log_e a^n . a^{mx}$
(iii) $1 + \log_e a^n . a^{mx}$ (iv) $n\log_e a^n . a^{mx}$

(b) If $u = x^m . y^n$, then $\frac{\partial^2 u}{\partial^2 x^2}$ is

(i) $m(m-1)x^{m-2}y^n$ (ii) $n(n-1)x^m y^{n-2}$
(iii) $(m-1)x^{m-1}y^n$ (iv) $(n-1)x^m y^{n-2}$

(c) $\int \cos(ax+b)dx$ *is*

(i) $\frac{1}{a}sin(ax+b)$ (ii) $\frac{1}{a}cosax$
(iii) $sin(ax+b)$ (iv) $\frac{1}{2}cos(ax+b)$

(d) $\int a^x\, dx$ is

(i) $\frac{a^x}{\log_e a}$ (ii) $\frac{\log_e a}{a^x}$ (iii) $\frac{e^x}{\log_e ax}$ (iv) $\frac{\log_e x}{a^x}$

(e) Let $\vec{r} = \vec{r_1} + \vec{r_2}$, then $\frac{d\vec{r}}{dt}$ is equal to

(i) $\frac{d\vec{r_1}}{dt} + \frac{d\vec{r_2}}{dt}$ (ii) $\frac{d\vec{r_1}}{dt} + \frac{d\vec{r_2}}{dt}$
(iii) $\vec{r_1}.\frac{d\vec{r_2}}{dt} + \vec{r_2}.\frac{d\vec{r_1}}{dt}$ (iv) $\vec{r_1}.\frac{d\vec{r_2}}{dt} - \vec{r_2}.\frac{d\vec{r_1}}{dt}$

(f) If $\vec{r} = \vec{a}cos\omega t + \vec{b}sin\omega t$, then $\frac{d^2\vec{r}}{dt^2}$ is

(i) $-\omega^2 r$ (ii) $\frac{\omega^2}{r}$ (iii) $-\frac{\omega}{r^2}$ (iv) $-\omega^4 r^2$

(g) If by change of axes without change of origin the expression
$ax^2 + 2hxy + by^2$ Becomes $a_1x_1^2 + 2h_1x_1y_1 + b_1y_1^2$ then

(i) $a + b = a_1 + b_1$ (ii) $ab = a_1b_1$
(iii) $a - b = a_1 - b_1$ (iv) $\frac{a}{b} = \frac{a_1}{b_1}$

(h) The equation of the directrix of the parabola

$$y^2 = 4ax \text{ is}$$

(i) $x+a=0$ (ii) $x=a$ (iii) $x=0$ (iv) $x-a=0$

(*i*) The sets {x:a<x≤b} is a

(i) Close interval (ii) Open interval
(iii) Half-open interval (iv) None of those

(j) $\sum\frac{1}{\eta^p}$ is convergent if

(i) $p>1$ (ii) $p<1$ (iii) $p = 1$ (iv) $p \le 1$

Section-II (marks: 90)

Each question has two part (a) carries 2 marks and part (b) carries 7 marks.
Answer ten questions, selecting at least one from each Group.

Group-A

2. (a) To find y_n when y=$sin(ak+b)$.
(b) State and prove Leibniz's theorem.

3. (a) Prove that $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ to ∞.
(b) If $= \frac{x^2y^2}{x+y}$, show $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} = 3u$.

4. (a) Show that the part of the tangent to $xy = c^2$ included between the co-ordinates axes is bisected at the point.
(b) To find the radius of curvature at any point (x,y) on the Cartesian curve Y=$f(x)$.

5. (a) Find the asymptotes of the curve

$$(x^2 - y^2)(x + 2y + 1) + x + y + 1 = 0.$$

(b) Find the maxima and minima of the function

$$x^3 + y^3 - 12x - 3y + 20.$$

Group-B

6. (a) Integrate $\int \frac{\sin^2\theta}{(1+\cos\theta)^2}d\theta$
(b) Integrate $\int \frac{x^2+1}{x(x^2-1)}dx$

7. (a) Evaluate $\int_0^{\pi} \frac{xdx}{1+\cos^2 x}$
(b) Show that $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \frac{\pi}{4}$.

8. (a) Find the area of the loop of the curve

$$Y^2 = x(x-1)^2.$$

(b) Find the area enclosed by the parabola

$$y^2 = 4ax \quad and \quad x^2 = 4by.$$

9. (a) The circle $x^2 + y^2 = a^2$ revolves round the x-axis. Find the surface and volume of the solid generated.
(b) Find the area of the surface formed by the revolution of $x^2 + 4y^2 = 16$ about its major axis.

Group-C

10.(a) Define differentiation of vector product of two vectors.
(b) If $\vec{r_1} = t^2\vec{i} - t\vec{j} + (2t+1)\vec{k}$, $\vec{r_2} = (2t-3)\vec{i} + \vec{j} - t\vec{k}$
Find (i) $\frac{d}{dt}(\vec{r_1}.\vec{r_2})$ (ii) $\frac{d}{dt}(\vec{r_1} \times \vec{r_2})$ when $t = 1$.

11.(a) Prove that $\nabla.(\vec{a} \pm \vec{b}) = \nabla.\vec{a} \pm \nabla.\vec{b}$
(b) State and prove that the curl of a vector field.

12.(a) Prove that $\nabla.(\vec{a} \times \vec{r}) = \vec{r}.\nabla \times \vec{a}$
(b) Find div (curl $\vec{F}$) where $\vec{F} = x^2y\vec{i} + xz\vec{j} + 2yz\vec{k}$.

Group-D

13.(a) Transform the equation
$4x^2 + 3y^2 - 2xy + 3x - 7\ y + 5 = 0$to parallel axes through the point (4,-1).
(b) Find the vertex, focus, axes and Latus rectum of the parabola $4y^2 + 12x - 20y + 67 = 0$.

14.(a) Find the eccentricity of an ellipse if its Latus rectum is equal to one-half of its major axis.
(b) To find the equation of a tangent to the conic, $S \equiv ax^2+2hxy+by^2+2gx+2fy+c=0$ at the point(x_1, y_1).

15.(a) To find the condition that the line $Y = mx + c$ may touch the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
(b) To prove that from a point (x_1, y_1) there are three normals can be drawn to a parabola $y^2 = 4ax$.

16.(a) Define chord of contact.
(b) To find the polar equation of the normal at any point 'α' of a conic $l/_r = 1 + e\cos\theta$, the focus being the pole.

Group-E

17.(a) Define closed set and open set in R.
(b) Prove that the intersection of two open sets in R is open.

18.(a) Define Cauchy sequences.
(b) The intersection of an arbitrary collection of closed set is closed.

19.(a) Prove that every convergent sequence is a Cauchy sequence.
(b) Prove that a set is closed iff its complement is open.

20.(a) State d'Alembert ratio test.
(b) State and prove Pringsheim's theorem.

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YEAR: - 2012 by pankaj

SUB: - MATH (sub) F.M:- 100

Section-I (Compulsory) (marks: 10)

1. Select the *correct* answer in the following: 1X10

(a) $D^n\left(\frac{1}{x+a}\right) =$

(i) $\frac{n!}{(x+a)^2}$ (ii) $\frac{n!}{(x+a)^{n+1}}$ (iii) $\frac{(-1)^n n!}{(x+a)^{n+1}}$ (iv) $\frac{(-1)^n n!}{(x+a)^n}$

(b) $\int \frac{dx}{a^2+x^2} =$

(i) $\tan^{-1}\frac{x}{a}$ (ii) $\frac{1}{a}\tan^{-1}\frac{x}{a}$ (iii) $\tan^{-1}\frac{a}{x}$ (iv) $\frac{1}{a}\tan^{-1}\frac{a}{x}$

(c) The radius of curvature ρ of a Cartesian curve is equal to

(i) $\frac{(1+y_1)^{\frac{3}{2}}}{y_2}$ (ii) $\frac{(1+y_1{}^2)^{\frac{2}{3}}}{y_2}$ (iii) $\frac{(1+y_1{}^2)^{\frac{3}{2}}}{y_2}$ (iv) $\frac{(1+y_1)^{\frac{2}{3}}}{y_2}$

(d) $\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}dx =$

(i) $\frac{\pi}{2}$ (ii) $\frac{\pi}{3}$ (iii) $\frac{\pi}{4}$ (iv) 2π

(e) General equation of conic

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

will represent a circle of

(i) $a=b, h>0$ (ii) $a=b, h=0$

(iii) $a\neq b, h=0$ (iv) $a=b, h<0$

(f) The condition that the line $y=mx+c$ may touch the parabola is

(i) $c=\frac{a}{m}$ (ii) $c=\frac{m}{a}$ (iii) $c=a^2/m$ (iv) $c=m^2/a$

(g) If $\vec{a}$ is a vector of constant magnitude, then which one is correct statement

(i) $\vec{a}.\frac{d\vec{a}}{dt}=0$ (ii) $\vec{a}\times\frac{d\vec{a}}{dt}=\vec{0}$ (iii) $\frac{d\vec{a}}{dt}=\vec{0}$ (iv) None of those

(f) If $\vec{r_1}=\vec{l}+2t\,\vec{j}-3t\,\vec{k}$ $\vec{r_2}=-t\,\vec{l}+\vec{j}+\vec{k}$

then $\frac{d}{dx}(\vec{r_1}.\vec{r_2})=$

(i) – 1 (ii) – 2 (iii) – 3 (iv) 0

(*i*) The series $1+\frac{1}{2}+\frac{1}{3}+\cdots$ to ∞ is

(i) convergent (ii) divergent

(iii) oscillatory (iv) None of those

(*j*) The sequence $1, 2, 3, 4, \ldots$ to ∞ is

(i) divergent (ii) oscillatory

(iii)convergent (iv) None of those

Section-II (marks: 90)

Each question has two part (a) carries 2 marks and part (b) carries 7 marks.
Answer ten questions, selecting at least one from each Group.

Group-A

2. (a) Find the $\frac{d^2y}{dx^2}$ if $x=a\cos\theta, y=a\sin\theta$.

(b) If $y^{1/m}+y^{-1/m}=2x$

Prove that $(x^2-1)y_2+xy_1-m^2y=0$.

3. (a) If $u=\sin^{-1}\left(\frac{x}{a}\right)$

Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$.

4. (a) What the formula for calculating radius of curvature of parametric curve.

(b) Find the pedal equation of the curve $r=ae^{\theta\cot\alpha}$.

5. (a) Prove that $x-\sin x$ has neither maximum nor a minimum value.

(b) Find the real asymptotes of the curve

$$x^3+y^3=3axy.$$

Group-B

6. (a) Write the integer of $\int\sqrt{x^2-a^2}\,dx$.

(b) Integrate $\int\frac{dx}{1+x^3}$

7. (a) Evaluate $\int_0^{\pi/2}\sin^2 x\cos^5 x\,dx$.

(b) Find reduction formula for $\int \sin^n x\ dx$.

8. (a) Discuss the symmetrical aspect of the curve

$$x^2+y^2=a^2$$

(b) Find the length of the curve $6y^2=x^3$ from the origin to the point where $x=14$.

9. (a) If a, b are the ordinates of end points of the curve $y=f(x)$, let this curve revolves about x –axis. Write the formula for surface of revolution so generated.

(b) Find the volume of the solid generated by the revolution of cardioids $r=a(1+\cos\theta)$ about $\theta=0$.

Group-C

10. (a) State the geometrical meaning of $\frac{d\vec{r}}{dt}$ where $\vec{r}$ is the position vector of any point on a curve.

(b) If $\vec{r}=\vec{a}\cos\omega t+\vec{b}\sin\omega t$ show that $\vec{r}\times\frac{d\vec{r}}{dt}=w\vec{a}\times\vec{b}$.

11.(a) Prove that necessary and sufficient condition for a scalar point function to be constant is that $\nabla\emptyset=0$.

(b) Find the unit vector normal to the surface $z=x^2+y^2$ at the point $(-1,-2,1)$.

12. (a) Interpret the physical meaning of $\nabla\times\vec{F}$.

(b) Prove that $\nabla\times(\nabla\emptyset)=0$.

Group-D

13. (a) Transform the equation

$x^2+y^2+2(x-y)+1=0$ when axes are rotated through an angle of 45° without shifting the origin.

(b) What conic section is represented by

$$2x^2+3y^2-4x-12y+13=0$$

Find its centre, axes and eccentricity.

14. (a) Find the equation of the tangent to the parabola $y^2=4ax$ at the point (x_1,y_1).

(b) Find the equation of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $(a\cos\theta, b\sin\theta)$.

15. (a) Define pole and polar line.

(b) Find the equation of the chord of contact of tangent drawn from the point (x_1,y_1) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

16. (a) Find the polar equation of a circle whose centre is $(c,\propto)$ and radius is a.

(b) Find the equation of pair of tangents drawn from a point the conic $l/r=1+e\cos\theta$.

Group-E

17. (a) Prove that interior of a set is an open set.

(b) Prove that union of finite number of closed sets is also closed.

18.(a) Give an example which is an open set but not an interval.

(b) State and prove Cauchy's General Principle of convergence of sequence.

19. (a) Every Cauchy sequence in R is a convergent sequence.

(b) If $\lim_{n\to\infty}u_n=l$, $\lim_{n\to\infty}v_n=l'$, prove that $\lim_{n\to\infty}(u_n+v_n)=l+l'$

20.(a) Define convergent and oscillatory series.

(b) Examine the convergency and divergency of an series

$$\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots\ldots\ldots\ldots$$